Equations of Lines

To write the equation of a line, you need two pieces of information:

1) point
2) slope

set up form	$\Rightarrow$	point slope-form

$$y - y_1 = m(x - x_1)$$

$(x_1, y_1) = $ point
$m = $ slope

Answer forms	$\Rightarrow$	standard form

$$ax + by = c$$

get x & y's on the same side.

Slope - intercept form

$$y = mx + b$$

Solve for y

Start here

need: point slope

point-slope
$$y - y_1 = m(x - x_1)$$

finish here

standard form
$$ax + by = c$$

OR

slope-intercept form
$$y = mx + b$$

ANSWER FORM $\left(\begin{array}{c} a > 0 \\ a \neq \text{fraction} \end{array}\right)$

ANSWER & INFORMATION FORM
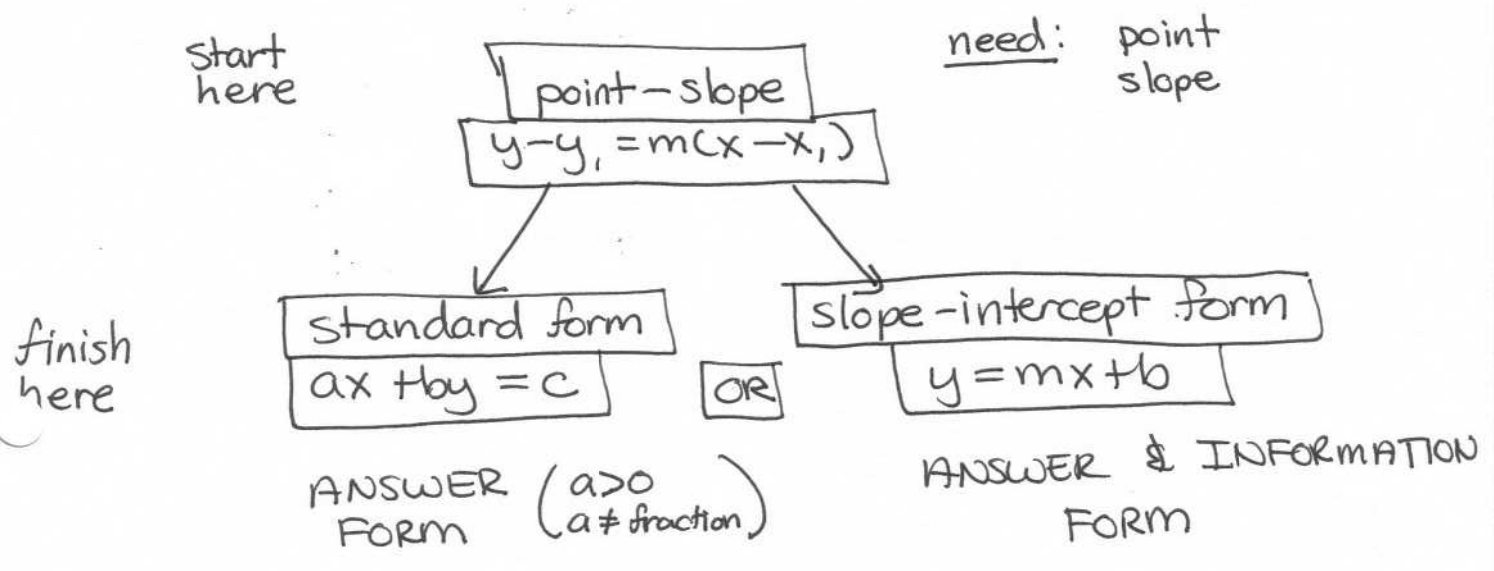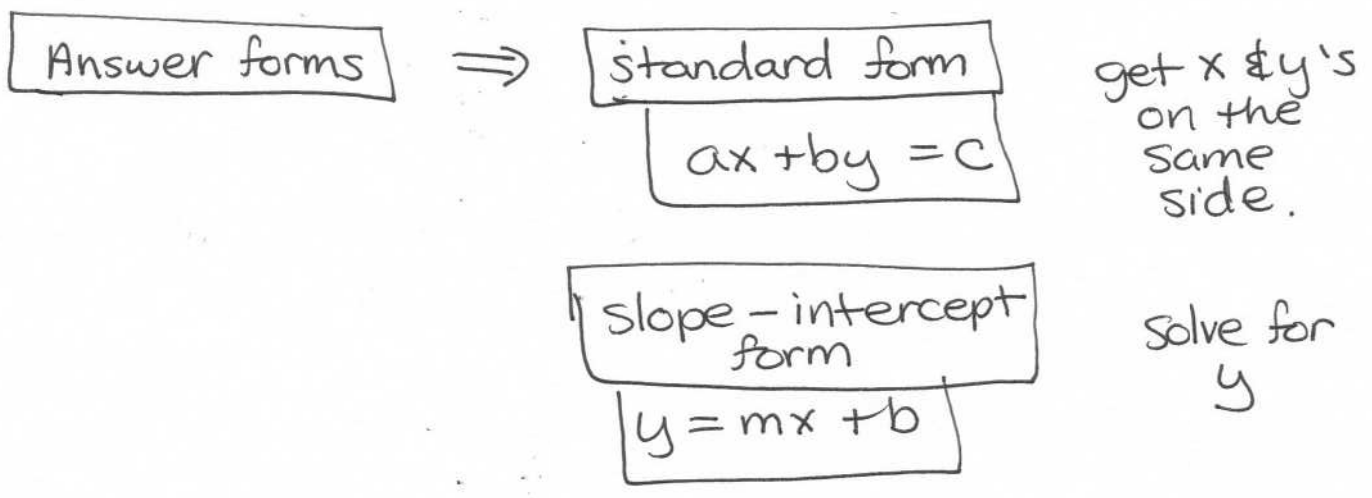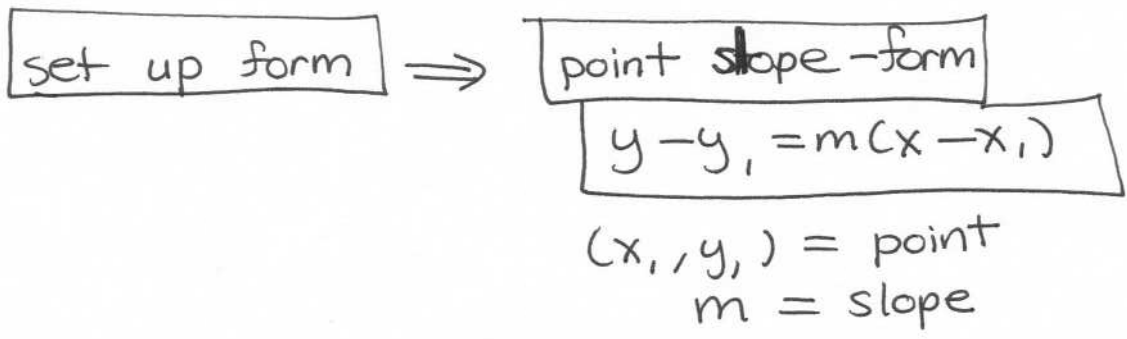
Example1

Write the equation of the line through points $(-2,5)$ and $(4,8)$. Write your answer in standard form.

need: point $(-2,5)$ or $(4,8)$ is ok

slope $m = \dfrac{8-5}{4--2} = \dfrac{3}{6} = \dfrac{1}{2}$

I'll pick $(-2,5) = (x_1, y_1)$

$m = \dfrac{1}{2}$

set up:

$y - y_1 = m(x - x_1)$

$y - 5 = \dfrac{1}{2}(x --2)$

$y - 5 = \dfrac{1}{2}x + 1$

for standard form → get x & y's on same side → multiply by lcd first

$lcd = 2$

$2y - 5(2) = \dfrac{1}{2}(2)x + 2(1)$

$2y - 10 = x + 2$

$-x + 2y = 2 + 10 = 12$ ← multiply by -1

$\boxed{x - 2y = -12}$ ($a \neq$ negative)

Example 2

Write the equation of the vertical line through $(-1, 7)$.

vertical line $\Rightarrow x = c$

so $\boxed{x = -1}$

Example 3

Write the equation of the line through $(-2, 5)$ and $(8, -9)$. Write your answer in slope-intercept form.

__need__ point: $(-2, 5)$ or $(8, -9)$ either is ok

slope: $m = \dfrac{-9-5}{8--2} = \dfrac{-14}{10} = \dfrac{-7}{5}$

point: $(8, -9) = (x_1, y_1)$
slope: $m = \left(\dfrac{-7}{5}\right)$

Set up
(start)

$$y - y_1 = m(x - x_1)$$
$$y + 9 = -\tfrac{7}{5}(x - 8)$$
$$y + 9 = -\tfrac{7}{5}x + \tfrac{56}{5}$$

$\swarrow \dfrac{9}{1}$

Slope-intercept
form
$\left(\begin{array}{c}\text{solve for} \\ y\end{array}\right)$

$$y = -\tfrac{7}{5}x + \tfrac{56}{5} - 9$$
$$y = -\tfrac{7}{5}x + \tfrac{56}{5} - \tfrac{45}{5}$$

$$\boxed{y = -\tfrac{7}{5}x + \tfrac{11}{5}}$$

Equations of Lines involving parallel & perpendicular cases

Example 1

Write the equation of the line through $(+1, 5)$ that is parallel to the line $y = 2x + 7$. Write your answer in slope-intercept form.

__need__ point $(+1, 5)$

slope $//$ to $y = \textcircled{2}x + 7$ $\Rightarrow$ the same slope as that line

so $(-1, 5)$ $y - y_1 = m(x - x_1)$

$m = 2$ $y - 5 = 2(x - +1)$

$y - 5 = 2x \pm 2$ (solve for y)

$\boxed{y = 2x + 3}$

Example 2

Write the equation of the Line through $(5, -8)$ that is perpendicular to the line $6x - 3y = 7$. Write your answer in standard form.

need: $\boxed{\text{point}}$ $(5, -8)$

$\boxed{\text{slope}}$ $\perp$ to slope from   need $\boxed{m = \frac{-1}{2}}$ the opp. reciprocal of 2

$6x - 3y = 7$

$6x - 3y = 7$

$-3y = -6x + 7$

$y = \frac{-6}{-3}x + \frac{7}{-3}$

$y = \textcircled{2}x - \frac{7}{3}$

$y - y_1 = m(x - x_1)$

$y + 8 = -\frac{1}{2}(x - 5)$

$y + 8 = -\frac{1}{2}x + \frac{5}{2}$ get x & y on same side

lcd $= 2$

$2y + 16 = -x + 5$

$x + 2y = 5 - 16$ $\boxed{x + 2y = -11}$

Equations of Lines — vertical & horizontal lines

note : // to vertical line is another vertical line

⊥ to vertical line is a horizontal line

| vertical | $x = c$ |
| horizontal | $y = c$ |

Example 1

Find the equation of the line through $(-6, 8)$ that is perpendicular to the line $y = 3$.

answer : $\boxed{x = -6}$

Example 2

Find the equation of the horizontal line through $(-1, 3)$.

answer : $\boxed{y = 3}$.

Example 3

Find the equation of the line through $(-5, 12)$ that is parallel to the line $x = 6$.

answer: $\boxed{x = -5}$